Week 4: Mass-spring loudspeaker model, transient analysis, acoustic domain

Microphone and Loudspeaker Design - Level 5

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What are we covering today?

1. Mass-spring model of loudspeaker
2. Transient and steady state analysis
3. Acoustic domain

A weekly fact about Salford..!

Did you know...

- Both Karl Marx and Friedrich Engels spent time in Salford, studying the plight of the British working class. In his book 'The Condition of the Working Class in England' (1844), Engels described Salford as "...a very unhealthy, dirty and dilapidated district." Alongside Marx, Engles co-authored The Communist Manifesto, which has sold over 500 million copies, making it one of the four best-selling books of all time.

Mass-spring model of loudspeaker

Impedance analogy: mass-spring-damper

- We have been considering the mass-spring-damper system - why?
- Impedance analogy:

$$F \to V \qquad u \to I \tag{1}$$

- Mobility analogy:

$$F \to I \qquad u \to V \tag{2}$$

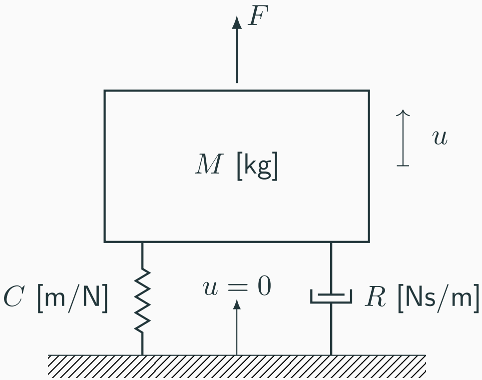

Figure 1: Mass-spring-damper.

Equivalent circuits: impedance vs. mobility

- Impedance:

$$F \to V \qquad u \to I \tag{3}$$

- Mobility:

$$F \to I \qquad u \to V \tag{4}$$

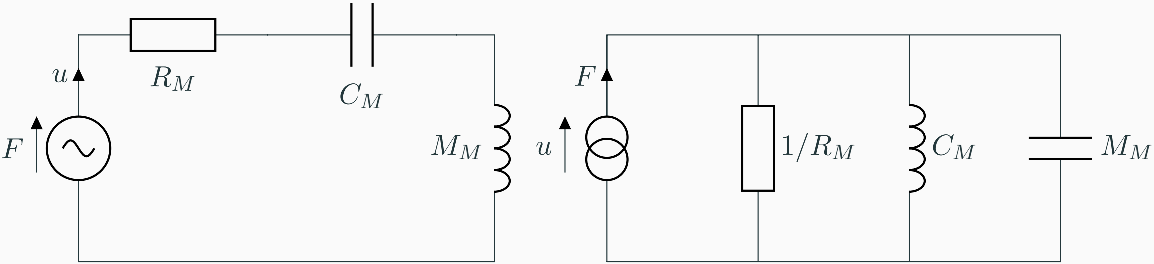

Figure 2: Equivalent circuits: impedance vs. mobility

Loudspeaker diaphragm: free body diagram

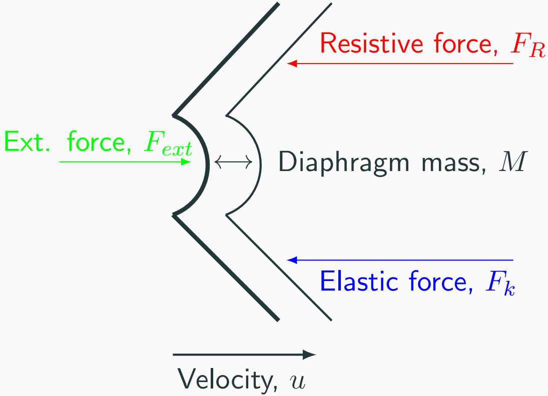

Figure 3: Loudspeaker free body diagram

- How can we model a loudspeaker diaphragm? First consider a force balance.
- According to lumped parameter assumptions:
  - loudspeaker diaphragm as a single mass element
  - spider and surround as a lumped spring with a viscous damper
- So it turns out our mass on a spring model actually describes the dynamics of a loudspeaker diaphragm!

Loudspeaker diaphragm: free body diagram

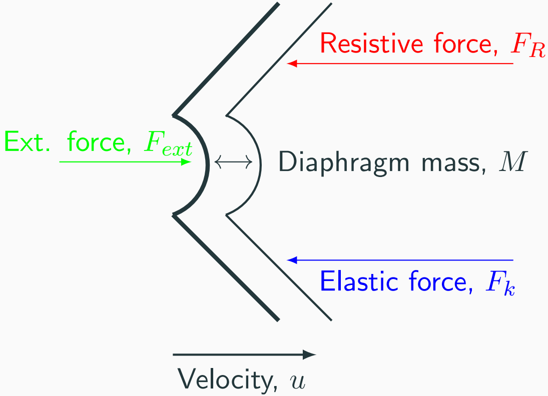

Figure 3: Loudspeaker free body diagram

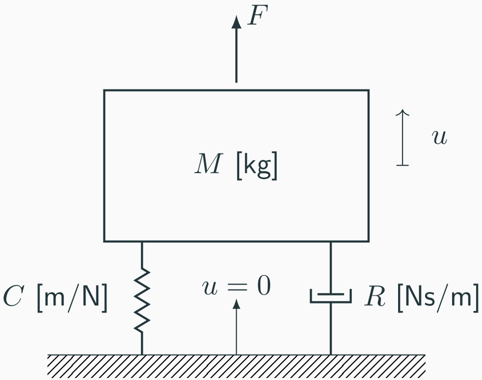

Figure 4: Mass-spring-damper.

Loudspeaker diaphragm: free body diagram

- We can model a loudspeaker driver as an equivalent circuit.

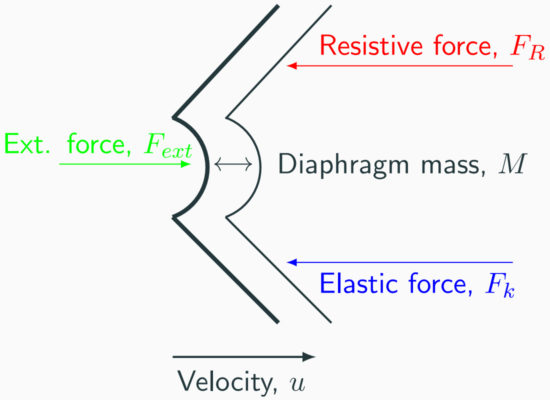

Figure 3: Loudspeaker free body diagram

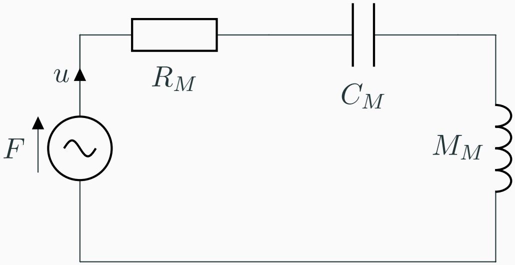

Figure 5: Equivalent impedance circuit

- Still need to consider the acoustic and electromagnetic domains...

Transient and steady state analysis

Mass-spring-damper: equation of motion

- We have analysed the dynamics of a mass-spring-damper system using an equivalent circuit approach
- Now we will consider a more conventional approach based on laws of classical mechanics
- Newton's 2nd Law:

$$\sum_i F_i = Ma = M\frac{d^2x}{dt^2} \qquad (5)$$

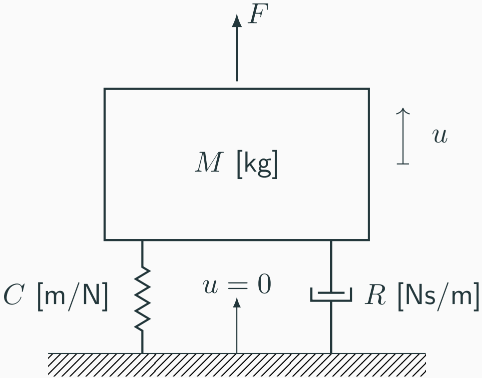

Figure 6: Mass-spring-damper.

$$-kx - R\frac{dx}{dt} + F_{ext} = M\frac{d^2x}{dt^2} \quad \rightarrow \quad F_{ext} = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2} \qquad (6)$$

Equation of motion: general and homogenous form

$$\underbrace{F_{ext} = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2}}_{\text{General form}} \qquad \underbrace{0 = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2}}_{\text{Homogenous form}} \tag{7}$$

- The general solution is made up of a **complementary** function plus the **particular** integral:

$$x = \overbrace{x_p}^{\text{Steady-state solution}} + \underbrace{x_{cf}}_{\text{Transient solution}} \tag{8}$$

- The transient solution found from homogenous form where $F = 0$.
- The steady state solution is found directly from general form.
- Great video on solving 2nd order differential equations: click this link!

Equation of motion: transient solution

$$\underbrace{F_{ext} = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2}}_{\text{General form}} \qquad \underbrace{0 = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2}}_{\text{Homogenous form}} \tag{9}$$

On the board...

Equation of motion: transient solution

Equation of motion: transient solution

Equation of motion: transient solution

Equation of motion: transient solution

Equation of motion: transient solution

- Our transient solution is

$$x_{cf} = A_1 e^{\frac{-R+\sqrt{R^2-4Mk}}{2M}t} + A_2 e^{\frac{-R-\sqrt{R^2-4Mk}}{2M}t} \tag{10}$$

- Factor out common exponential

$$x_{cf} = \underbrace{e^{-\frac{R}{2M}t}}_{\text{Decay}} \overbrace{\left[A_1 e^{\frac{\sqrt{R^2-4Mk}}{2M}t} + A_2 e^{\frac{-\sqrt{R^2-4Mk}}{2M}t}\right]}^{\text{Potential oscillation}} \tag{11}$$

- Three conditions:
- $4Mk < R^2$ Square root is positive - real roots - no oscillation (**over damped**)
- $4Mk = R^2$ Square root is 0 - decay term only - no oscillation (**critically damped**)
- $4Mk > R^2$ Square root is negative - complex roots – oscillation (**under damped**)
- Recalling $Q = \sqrt{Mk}/R$ the above are equivalent to: $Q > 1/2$, $Q = 1/2$ and $Q < 1/2$

Q factor vs. oscillation

$$x_{cf} = e^{-\frac{R}{2M}t}\left[A_1 e^{\frac{\sqrt{R^2-4Mk}}{2M}t} + A_2 e^{\frac{-\sqrt{R^2-4Mk}}{2M}t}\right] \tag{12}$$

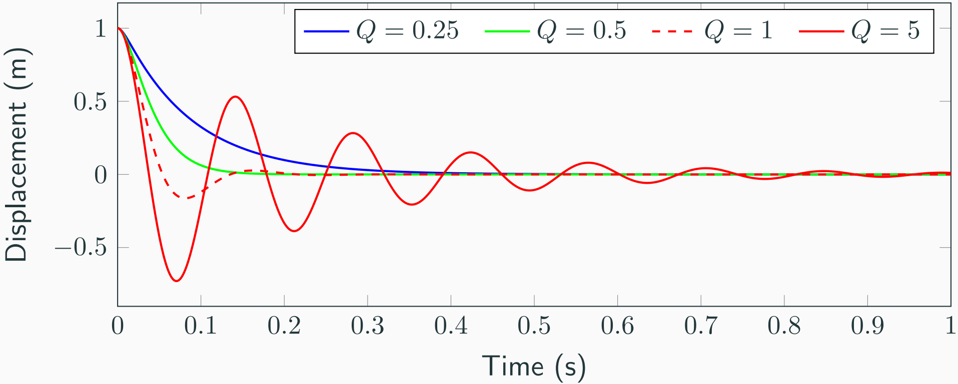

Figure 7: Over, under and critically damped oscillation - $x(0) = 1$ and $\dot{x}(0) = 0$

Q factor vs. peakyness

$$u = \frac{F}{R + j\omega m + \frac{k}{j\omega}} = \frac{F}{\frac{\sqrt{mk}}{Q} + j\omega m + \frac{\omega_c^2 m}{j\omega}} = \frac{F}{\frac{\sqrt{mk}}{Q} + j\omega m\left(1 - \frac{\omega_c^2}{\omega^2}\right)} \tag{13}$$

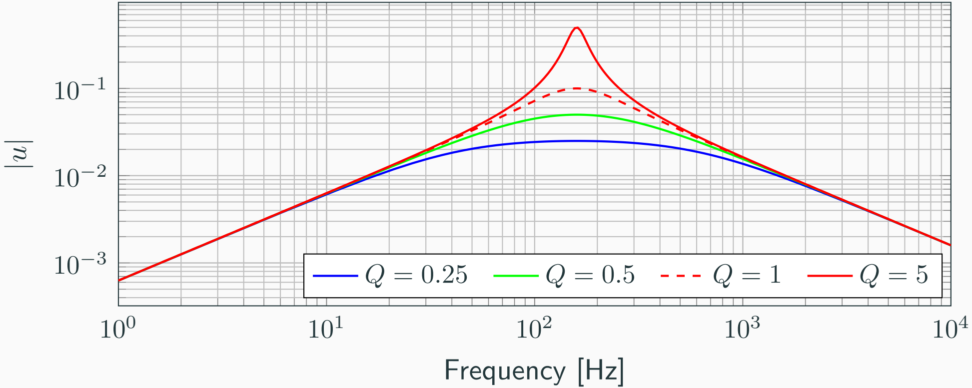

Figure 8: Frequency response of over, under and critically damped mass-spring-damper.

Equation of motion: steady-state solution

$$\underbrace{F_{ext} = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2}}_{\text{General form}} \qquad \underbrace{0 = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2}}_{\text{Homogenous form}} \tag{14}$$

- To get the steady state solution we consider: $F_{ext} = F_0 e^{j\omega t}$
- Linear equation, so response will also be periodic: $x_p = x_0 e^{j\omega t}$

$$F_0 e^{j\omega t} = \left((j\omega)^2 M + j\omega R + k\right) x_0 e^{j\omega t} \tag{15}$$

- Steady state solution given by:

$$x_p = \frac{F_0 e^{j\omega t}}{(j\omega)^2 M + j\omega R + k} = \frac{F_0 e^{j\omega t}}{j\omega \underbrace{\left(R + j\left[\omega M - \frac{k}{\omega}\right]\right)}_{\text{Impedance } Z}} \tag{16}$$

Equation of motion: complete solution

- The general solution is the complementary function plus the particular integral:

$$x(t) = x_p + x_{cf} \tag{17}$$

$$x(t) = \frac{F_0 e^{j\omega t}}{j\omega\left(R + j\left[\omega M - \frac{k}{\omega}\right]\right)} + e^{-\frac{R}{2M}t}\left[A_1 e^{\frac{\sqrt{R^2-4Mk}}{2M}t} + A_2 e^{\frac{-\sqrt{R^2-4Mk}}{2M}t}\right] \tag{18}$$

- As $t \to \infty$ the transient part of the solution tends to zero.
- **We will focus on the steady state part.**

Acoustic domain

Acoustic impedance

- Have already covered electrical and mechanical impedance...
- Electrical impedance (opposition to flow of current)

$$Z_E = \frac{V}{I} \tag{19}$$

- Mechanical impedance (opposition to mechanical motion)

$$Z_M = \frac{F}{u} \tag{20}$$

- Acoustic impedance is more awkward - there are three different types...

Acoustic impedance: three types

- Acoustic impedance describes the opposition to motion or flow of air. Acoustic impedance relates the acoustic pressure at a surface to the velocity.
- **Specific** acoustic impedance

$$z_A = \frac{p}{u} \qquad u \text{ is the particle velocity} \tag{21}$$

- **Acoustic** impedance

$$Z_A = \frac{p}{U} \qquad U \text{ is the volume velocity} \tag{22}$$

- **Radiation** impedance

$$Z_{Ar} = S z_A \qquad S \text{ is vibrating surface area} \tag{23}$$

Acoustic impedance: volume velocity

- The volume velocity is the product of the component of particle velocity u normal to a vibrating surface and the differential surface area:

$$dU = \hat{n} \cdot u dS \tag{24}$$

- For a uniformly vibrating surface area S we have

$$U = uS \tag{25}$$

- Has units of [m^3/s] hence the name *volume* velocity
- Volume velocity is a scalar (not a vector like particle velocity)

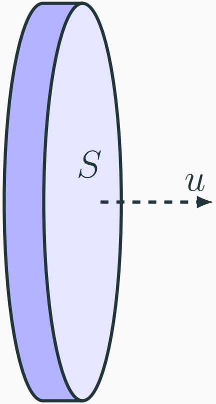

Figure 9: Volume velocity of a rigid piston.

Acoustic impedance: relation to mechanical impedance

- Acoustic impedance

$$Z_A = \frac{p}{U} \qquad U \text{ is the volume velocity} \tag{26}$$

- Mechanical impedance

$$Z_M = \frac{F}{u} \qquad u \text{ is the surface velocity} \tag{27}$$

- Recalling that $p = F/S$ and $U = uS$

$$Z_A = \frac{p}{uS} = \frac{F/S}{uS} = \frac{F}{uS^2} = \frac{Z_M}{S^2} \tag{28}$$

- Acoustic and mechanical impedance are related by factor of $1/S^2$

Next week...

- Acoustic domain - basic elements and equivalent circuits
- Transducers (electro-mechanical, mechano-acoustical)
- Ideal transformers

- Reading:
  - Acoustic domain: lecture notes, chp. 4 (all)
  - Coupling domains: lecture notes, sec. 6.1-6.5 (all)